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LIE THEORY AND APPLICATIONS. II

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Personal development

Josef Teichmann, who was supported by this project for 2 months only, but was supported by the predecessor of this project, had his 'Habilitation' approved at TU Vienna in 2002. He is a. Prof. at TU Vienna now.

Armin Rainer (fully supported by this project) had his PhD thesis approved in February 2004, and will be promoted sub auspiciis praesidentis rei publicae on March 12, 2005.

Stefan Haller's (1/2 post-doc in this project - the other half came from the University of Vienna) 'Habilitation' is in the final stages of approval now. He will take a temporary position at the MPI for Mathematics in Bonn in February 2005.

Simon Hochgerner's (fully supported by this project) PhD thesis is in the final stages of preparation and will be finished in the first quarter of 2005.

Mona Linkmann's (fully supported for 7 months) work on her PhD thesis stopped by her own wish.

Description of the scientific work

Invariant Theory I: Lifting of curves. Collaboration with Dmitri Alekseevsky from Hull (UK) and Moscow, Mark Losik from Saratov (RU), Andreas Kriegl and Armin Rainer from Vienna.

In the paper [M65] which was prepared in the project P 10037 we investigated the following problem: Let

$$P(t)(x) = x^n - \sigma_1(t)x^{n-1} + \cdots + (-1)^n\sigma_n(t)$$

be a polynomial with all roots real, smoothly parametrized by t near 0 in $\mathbb{R}$. Can we find n smooth functions $x_1(t), \dots, x_n(t)$ of the parameter t defined near 0, which are the roots of $P(t)$ for each t? We showed that this is possible under quite general conditions: real analyticity or no two roots should meet of infinite order. Some applications to perturbations of unbounded operators in Hilbert space are also given.

When this paper was published we received an email by two French mathematicians, Jaques Chaumat and Anne-Marie Chollet, who found out that counterexample 2.5 in our paper cited above contradicts a result by Bronshtein, who showed in 1979 that for a C^n-curve of polynomials one can always choose the roots differentiable, with bounded derivatives. Counterexample 2.5 is wrong, and we were able to combine Bronshtein's estimates with the methods of our paper to show that for a C^{2n}-curve of polynomials the roots can be arranged to be C^1, and for a C^{3n}-curve even twice differentiably. This is published in [M91].

As an application of the results in [M91] and of convenient calculus we could show in [M89] a strong result in perturbation theory of unbounded operators, namely:

Let $t \mapsto A(t)$ for $t \in \mathbb{R}$ be a curve of unbounded self-adjoint operators in a Hilbert space with common domain of definition and with compact resolvent.

(A) *If $A(t)$ is real analytic in $t \in \mathbb{R}$, then the eigenvalues and the eigenvectors of $A(t)$ may be parameterized real analytically in t.*
(B) *If $A(t)$ is C^∞ in $t \in \mathbb{R}$ and if no two unequal continuously parameterized eigenvalues meet of infinite order at any $t \in \mathbb{R}$, then the eigenvalues and the eigenvectors can be parameterized smoothly in t, on the whole parameter domain.*
(C) *If A is C^∞, then the eigenvalues of $A(t)$ may be parameterized twice differentiably in t.*
(D) *If $A(t)$ is $C^{1,\alpha}$ for some $\alpha > 0$ in $t \in \mathbb{R}$, then the eigenvalues of $A(t)$ may be parameterized in a C^1 way in t.*

Part (A) is due to Rellich in 1940, Part (B) has been proved in the paper cited in the beginning, 7.8, in 1997; there we gave also a different proof of (A). Parts (C) and (D) are for the first time in [M89].

The problem of choosing roots differentiably can be reformulated in the following way: Let the symmetric group S_n act on $\mathbb{R}^n$ by permuting the coordinates (the roots), and consider the polynomial mapping $\sigma = (\sigma_1, \dots, \sigma_n) : \mathbb{R}^n \to \mathbb{R}^n$ whose components are the elementary symmetric polynomials (the coefficients). Given a smooth curve $c : \mathbb{R} \to \sigma(\mathbb{R}^n) \subset \mathbb{R}^n$, is it possible to find a smooth lift $\bar{c} : \mathbb{R} \to \mathbb{R}^n$ with $\sigma \circ \bar{c} = c$?

In the paper [M73] which was prepared in the project P 10037, we tackled the following generalization of this problem. Consider an orthogonal representation of a compact Lie group G on a real vector space V. Let $\sigma_1, \dots, \sigma_n$ be a system of homogeneous generators for the algebra $\mathbb{R}[V]^G$ of invariant polynomials on V. Then the mapping

$$\sigma = (\sigma_1, \dots, \sigma_n) : V \to \mathbb{R}^n$$

defines a bijection of the orbit space V/G to the semialgebraic set $\sigma(V) \subseteq \mathbb{R}^n$. A curve

$$c : \mathbb{R} \to V/G = \sigma(V) \subseteq \mathbb{R}^n$$

in the orbit space V/G is called smooth if it is smooth as a curve in $\mathbb{R}^n$. This is well defined, i.e. does not depend on the choice of generators.

Problem. Given a smooth curve $c : \mathbb{R} \to V/G$ in the orbit space, does there exist a smooth lift to V, i.e. a smooth curve $\bar{c} : \mathbb{R} \to V$ with $c = \sigma \circ \bar{c}$?

In the former project P 10037, in the paper [M73], we gave satisfactory answers under similar conditions (normal non-flatness) as 'no two roots meet of infinite order' in the polynomial case. Also real analytic curves can be lifted to real analytic curves.

In this project, in [M99], we could show that always (without any conditions) any sufficiently often (higher than all degrees of the σ_k's) differentiable curve can be lifted to a differentiable curve. Note that in the case of polynomials we got a twice differentiable lift which was best possible. In [M99] the result seemed unattainable. But in [M105], for finite groups, we got a twice differentibale lift, by constructing suitable polynomials and applying Bronshtein's estimates to them. The required degree of differentiability is quite high; for certain reflection groups we got lower degrees. The result also holds for polar representations. In general, we could show the existence of twice differentiable lifts only if the representation has a certain property (Bronshtein's result for each slice representation). We study stability of this property under functorial constructions of representations.

The PhD thesis of Armin Rainer, was prepared with support of this project, he will be promoted sub auspiciis presidentis in March 2005.

An enlarged and corrected version [R] of this thesis has been submitted to Springer Lecture Notes in Mathematics. It is a careful presentation of all results (sauf the newest in [M105]) discussed in this section, and also gives full and detailed proofs of the results of Bronshtein mentioned above, and another proof due to Wakabayashi of the same estimates.

Invariant Theory II. In paper [M88], for a representation of a finite group G on a complex vector space V we determine when a holomorphic $\binom{p}{q}$-tensor field on the principal stratum of the orbit space V/G can be lifted to a holomorphic G-invariant tensor field on V. This extends also to connections. As a consequence we determine those holomorphic diffeomorphisms on V/G which can be lifted to orbit preserving holomorphic diffeomorphisms on V. This in turn is applied to characterize complex orbifolds.

[M90] treats the following result: Let X be a smooth algebraic variety endowed with an action of a finite group G such that there exists a geometric quotient $\pi_X : X \to X/G$. We characterize rational tensor fields τ on X/G such that the *pull back* of τ is regular on X: these are precisely all τ such that $\operatorname{div}_{R_{X/G}}(\tau) \geq 0$ where $R_{X/G}$ is the *reflection divisor* of X/G and $\operatorname{div}_{R_{X/G}}(\tau)$ is the $R_{X/G}$-*divisor* of τ. We give some applications, in particular to a generalization of Solomon's theorem. In the last section we show that if V is a finite dimensional vector space and G a finite subgroup of $\mathrm{GL}(V)$, then each automorphism ψ of V/G admits a biregular lift $\varphi : V \to V$ provided that ψ maps the regular stratum to itself and $\psi_*(R_{X/G}) = R_{X/G}$. This can be viewed as extending field automorphism from a subfield to a field of rational functions; a first in commutative algebra. This last result needs results from [M88] which are carried over to algebraic geometry by a neat trick.

In [M93] (still unfinished, since coauthor Popov is still discontent with the algebraic geometric presentation), we prove a weak version of the first main theorem of invariant theory, namely, we describe a class of representations of a reductive algebraic group on a vector space V such that $\mathbb{C}[V^q]^G$ is the integral closure of $\mathbb{C}[V^q]^G_{\mathrm{pol}}$, the subring generated by all generalized polarizations, in $\mathbb{C}[V^q]$. For finite groups we prove stronger results.

In [M94] finally lifting problems for more than one parameters are looked at. We characterize those regular, holomorphic or formal maps into the orbit space V/G of a complex representation of a finite group G which admit a regular, holomorphic or formal lift to the representation space V. In particular, the case of complex reflection groups is investigated.

Completing infinitesimal actions of vector fields and Lie algebras. This concerns 3 papers [M83], [M92], [M96], and [M104] and it is based on ideas arising from the paper [M56] which was prepared in the project P 10037-MAT. Parts of the results were already done in Richard Palais' Thesis in 1955. This is joint work with Franz Kamber from the University of Illinois (who was supported by this project) and with Boris Khesin.

Let me describe the results now: For a manifold M with vector field X which is not complete there exists a universal completion $\bar{M}, \bar{X}$ with an (infinitesimally) equivariant embedding $(M, X) \to (\bar{M}, \bar{X})$, where $\bar{M}$ is a manifold which might be not Hausdorff, and where $\bar{X}$ is a vector field with a complete flow on $\bar{M}$. This is due to Palais and to [M83]. In [M92] this was extended to the case that M is infinite dimensional, but then one has to require that X admits a local flow. If X is a Hamiltonian field for a function f on a symplectic or Poisson manifold M, then the symplectic or Poisson structure and the Hamiltonian extend uniquely to $\bar{M}$. In this sense completely integrable systems go to completely integrable systems. As an example, the classical method of characteristics allowed us to identify the universal completion of Burgers' equation as a manifold of curves in $\mathbb{R}^2$.

If instead of one vector field a (finite dimensional) Lie algebra $\mathfrak{g}$ acts on M there exists also a universal completion $\bar{M}$ as topological space with the action of the simply connected Lie group G of $\mathfrak{g}$ on $\bar{M}$. Here $\bar{M}$ might be even not T_1, so only a very weak smooth structure is surviving on $\bar{M}$. But the construction of $\bar{M}$ can be done orbit wise: For each orbit the completion is a manifold of the form G/H where H is a (in general not closed) Lie subgroup of G. The orbit spaces $M/\mathfrak{g}$ and M/G are homeomorphic. Symplectic or Hamiltonian actions go to symplectic or Hamiltonian actions.

The paper [M104] is devoted to $\mathfrak{g}$-manifolds whose completions have good properties: They are orbifolds, or have proper actions. It is determined how these properties can be seen on the $\mathfrak{g}$-manifold itself.

Gerbes and foliated bundles. Since Franz Kamber was supported for several months by this project, we list also the paper [K1] of Johan Dupont and Franz Kamber on *Gerbes, Simplicial Forms and Invariants for Families of Foliated Bundles*. Here the notion of a gerbe with connection is reformulated in terms of the simplicial De Rham complex. In particular, the usual Chern-Weil and Chern-Simons theory is particularly well adapted to this setting and gives rise to 'characteristic gerbes' associated to families of bundles and connections. In turn this gives invariants of families of foliated bundles. A special case is the Quillen bundle associated to families of flat $SU(2)$-bundles.

Reflections groups on Riemannian manifolds. The paper [M97] deals with discrete groups G of isometries of a complete connected Riemannian manifold M which are generated by reflections, in particular those generated by disecting reflections. It is shown that these are Coxeter groups, and that the the orbit space M/G is isometric to a Weyl chamber C which is a Riemannian manifold with corners and certain angle conditions along intersections of faces. We can also reconstruct the manifold and its action from the Riemannian chamber and its equipment of isotropy group data along the faces. We also discuss these results from the point of view of Riemannian orbifolds. Further investigations on this topic are under way.

Lie theory and representation theory. The unpublished paper [M81] arose from an attempt to understand extension theory of Lie algebras - we found out later that all results are available in the literature. Since this is a coherent and complete description of all results and since it is widely used, we list it here.

[M83] contains the corresponding super Lie algebra version of this extension theory.

The paper [M87] deals with infinitesimally faithful representations of a reductive complex connected algebraic group G. Namely, it induces a dominant morphism Φ from the group to its Lie algebra $\mathfrak{g}$ by orthogonal projection in the endomorphism ring of the representation space. The map Φ identifies the field $Q(G)$ of rational functions on G with an algebraic extension of the field $Q(\mathfrak{g})$ of rational functions on $\mathfrak{g}$. For the spin representation of $\mathrm{Spin}(V)$ the map Φ coincides with the classical Cayley transform multiplied with a regular function. In general, properties of Φ are established and these properties are applied to deal with a separation of variables (Richardson) problem for reductive algebraic groups: Find $\mathrm{Harm}(G)$ so that for the coordinate ring $A(G)$ of G we have $A(G) = A(G)^G \otimes \mathrm{Harm}(G)$. As a consequence of a partial solution to this problem and a complete solution for $SL(n)$ one has in general the equality $[Q(G) : Q(\mathfrak{g})] = [Q(G)^G : Q(\mathfrak{g})^G]$ of the degrees of extension fields. Among other results, Φ yields (for the complex case) a generalization, involving generic regular orbits, of the result of Richardson showing that the Cayley map, when G is semisimple, defines an isomorphism from the variety of unipotent elements in G to the variety of nilpotent elements in $\mathfrak{g}$. In addition if G is semisimple the Cayley map establishes a diffeomorphism between the real submanifold of hyperbolic elements in G and the space of infinitesimal hyperbolic elements in $\mathfrak{g}$. Some examples are computed in detail.

[M103] is mainly a review of the results of paper [M87]. In the end the Cayley transform $\Phi : G \to \mathfrak{g} \cong \mathfrak{g}^*$ is used to pullback the canonical coadjoint Poisson structure to G. Since Φ is not everywhere a local diffeomorphism, the pullback structure acquires poles, is only a rational Poisson structure: but surprisingly it turns out to be an honest Dirac structure $D \subset TG \times_G T^*G$, without infinities.

Symplectic manifolds. [H4] is an overview article on properties of locally conformal symplectic manifolds. These are geometric structures very close to symplectic manifolds. More precisely, one has given a non-degenerate two form which locally is conformal to a symplectic structure. The article summarizes some earlier joint work with Rybicki on this subject and some results from my thesis. Among these are perfectness results for the group of diffeomorphisms preserving a locally conformal symplectic structure (generalizing results of Banyaga for the symplectic case), infinitesimal analogs (generalizing a result of Calabi), a Weinstein chart (exhibiting this group as a Fréchet–Lie group), Pursell-Shanks type results (generalizing results of Omori), Filipkiewicz type results (integrated version of Pursell–Shanks) and Weinstein–Marsden reduction for locally conformal symplectic manifolds.

In [H9] we studied symplectic manifolds from the point of view of symplectic Hodge theory introduced by Brylinski. The symplectic structure gives rise to a co-differential. In contrast to the Riemannian case one does not have ellipticity here. Still one can use the co-differential to define a canonic filtration on the deRham cohomology. One particular filtration space is Brylinski's space of harmonic cohomology classes — those having co-closed representatives. In [H9] we computed this filtration in terms of the cohomology ring. Particularly we get explicit formulas for the dimensions of these filtration spaces in terms of ranks of Lefschetz type mappings. From this we obtained a close relationship between the surjectivity of Lefschetz type mappings and properties of the filtration — generalizing a result of Mathieu which tells that a symplectic manifold satisfies the hard Lefschetz theorem if and only if every cohomology class has a harmonic representative. We also showed that Poincaré duality is nicely compatible with the filtration, which has several nice corollaries. E.g. one gets a generalization to general symplectic manifolds of the fact that on a closed Kähler manifold the Betti numbers in odd dimensions are even. Also we obtained a Künneth theorem for the filtration spaces. This can be considered as motivation for studying the whole filtration and not just the harmonic cohomology since the harmonic cohomology of a product in general depends on the whole filtration of the two factors. Finally we considered complex projective space blown up along symplectic submanifolds. These are symplectic manifolds according to a theorem of McDuff. We could completely describe the filtration of the blown up manifolds in terms of the filtration of the submanifold.

In [H10] we studied what is called c–splitting conjecture. The question was raised by Lalonde and McDuff, and reads as follows. Given a Hamiltonian fiber bundle, do we then necessarily have that the rational cohomology of the total space is the tensor product of the cohomology of the base with the cohomology of the fiber — additively. The general situation is still open. However, for special fibers, e.g. manifolds which satisfy the hard Lefschetz theorem, the answer is affirmative according to an old result of Blanchard. Also there is a deep theorem of Lalonde and McDuff which tells that the answer is also positive whenever the base is a CW complex of dimension less than four. In [H10] we used the latter theorem and methods from [H9] to show that the c–splitting conjecture holds whenever the fiber satisfies a weakened Lefschetz condition. More precisely, for a $2n$ dimensional fiber the condition is that the Lefschetz type mappings $H^{n+1-k} \to H^{n+1+k}$ are onto for all $k \geq 0$.

In [H13] we generalized methods from [H9] to Poisson manifolds. Particularly every Poisson manifold comes with a canonic filtration of its deRham cohomology. Still we find that this filtration is compatible with Künneth theorem and Poincaré duality. However, we are not yet able to compute this filtration in non-trivial examples. The plan is to use this filtration to obtain restrictions on the homotopy type of Poisson mappings. There might also be applications to the homotopy type of the Hamiltonian group of symplectic manifolds, via the clutching construction. More precisely, a homotopy class of a map into the Hamiltonian group gives rise to a Hamiltonian fiber bundle via the clutching construction. If the homotopy class becomes trivial when considered in the full group of diffeomorphisms the fiber bundle becomes trivial as a smooth fiber bundle. However, the Poisson structure on the total space need not be trivial, and the harmonic filtration might be able to detect that. This paper is still under construction.

In [H7] we developed Floer theory based on non-contractible one periodic orbits (in a fixed homotopy class γ) of a symplectic vector field. It is interesting to look at the torsion of the Floer complex. This can be non-trivial even though the cohomology vanishes for almost all homotopy classes γ. This paper is still under construction.

About the paper [M100]: Homology of a Lie algebroid structure on a vector bundle E over M are usually considered as homology of the corresponding Batalin-Vilkovisky algebra associated with a chosen generating operator ∂ for the Schouten-Nijenhuis bracket on multisections of E. The generating operators that are homology operators, i.e. $\partial^2 = 0$, can be identified with flat E-connections on $\bigwedge^{\text{top}} E$ or divergence operators (flat right E-connections on $M \times \mathbb{R}$). The problem is that the homology group depends on the choice of the generating operator (flat connection, divergence) and no one seems to be privileged. For instance, if a Lie algebroid on T^*M associated with a Poisson tensor P on M is concerned, then the traditional Poisson homology is defined in terms of the Koszul-Brylinski homology operator $\partial_P = [d, i_P]$. However, the Poisson homology groups may differ from the homology groups obtained by means of 1-densities on M. The celebrated modular class of the Poisson structure of Weinstein measures this difference. An analogous statement is valid for triangular Lie bialgebroids. The concept of a Lie algebroid divergence, so a generating operator, associated with a 'volume form', i.e. nowhere-vanishing section of $\bigwedge^{\text{top}} E^*$, is completely classical. Less-known seems to be the fact that we can use 'odd-forms' instead of forms (as did de Rham) with same formulas for divergence and that such nowhere-vanishing volume odd-forms always exist. The point is that the homology groups obtained in this way are all isomorphic, independently on the choice of the volume odd-form. This makes the homology of a Lie algebroid a well-defined notion. From this point of view the Poisson homology is not the homology of the associated Lie algebroid T^*M but a deformed version of the latter, exactly as the exterior differential $d^\phi \mu = d\mu + \phi \wedge \mu$ of Witten is a deformation of the standard de Rham differential. In this language, the modular class of a Lie algebroid morphism $\kappa : E_1 \to E_2$ covering the identity on M is defined as the class of the difference between the pull-back of a divergence on E_2 and a divergence on E_1, both associated with volume odd-forms. In the case when $\kappa : E \to TM$ is the anchor map, we recognize the standard modular class of a Lie algebroid, but it is clear that other (canonical) morphisms will lead to other (canonical) modular classes.

Riemannian orbit spaces and completely integrable systems. In [M85] we investigate the rudiments of Riemannian geometry on orbit spaces M/G for isometric proper actions of Lie groups on Riemannian manifolds. Minimal geodesic arcs are length minimising curves in the metric space M/G and they can hit strata which are more singular only at the end points. This is phrased as convexity result. The geodesic spray, viewed as a (strata-preserving) vector field on TM/G, leads to the notion of geodesics in M/G which are projections under $M \to M/G$ of geodesics which are normal to the orbits. It also leads to 'ballistic curves' which are projections of the other geodesics. In examples (Hermitian and symmetric matrices, and more generally polar representations) we compute their equations by singular symplectic reductions and obtain generalizations of Calogero-Moser systems with spin.

The PhD thesis of Simon Hochgerner, called *'Singular cotangent bundle reduction & spin Calogero-Moser systems'* is devoted to develop the symplectic underpinning of the examples computed in [M85]: Singular stratified cotangent bundle reduction, and degenerate integrability of such a system. The main results of this thesis which I expect to be finished soon, are contained in [Ho]. This paper develops a bundle picture for the case that the configuration manifold has only a single isotropy type, and gives a formula for the reduced symplectic form in this setting. Furthermore, as an application of this bundle picture Calogero-Moser systems with spin are considered which are associated to polar representations of compact Lie groups.

Infinite dimensional manifolds and Lie groups, shape theory and pattern recognition. The paper [M98] arose from the attempt to find the simplest Riemannian metric on the space of 2-dimensional 'shapes'. By a shape we mean a compact simply connected region in the plane whose boundary is a simple closed curve. By requiring that the boundary curve has various degrees of smoothness, we get not just one space but a whole hierarchy of spaces. All these spaces will include, however, a core, namely the space of all shapes with C^∞ boundary curves. We expect that the most natural shape spaces will arise as the completions of this core space in some metric, hence we take this core as our basic space. Note that it is the orbit space

$$B_e(S^1,\mathbb{R}^2) = \operatorname{Emb}(S^1,\mathbb{R}^2)/\operatorname{Diff}(S^1)$$

of the space of all C^∞ embeddings of S^1 in the plane, under the action by composition from the right by diffeomorphisms of the circle. The space $\operatorname{Emb}(S^1,\mathbb{R}^2)$ is a smooth manifold, in fact an open subset of the Fréchet space $C^\infty(S^1,\mathbb{R}^2)$, and it is the total space of a smooth principal bundle with base $B_e(S^1,\mathbb{R}^2)$. In fact, most of our results carry over to the bigger orbit space of immersions mod diffeomorphisms:

$$B_i(S^1,\mathbb{R}^2) = \operatorname{Imm}(S^1,\mathbb{R}^2)/\operatorname{Diff}(S^1).$$

This action is not quite free hence this orbit space is an orbifold and not quite a manifold. This spaces are relevant for **computer vision**. To understand an image of the world, one needs to identify the most salient objects present in this image. In addition to readily quantifiable properties like color and area, objects in the world and their projections depicted by 2D images possess a 'shape' which is readily used by human observers to distinguish, for example, cats from dogs, BMW's from Hondas, etc. In fact people are not puzzled by what it means to say two shapes are *similar* but rather find this a natural question. This suggests that we construct, on some crude level, a mental metric which can be used to recognize familiar objects by the similarity of their shapes and to cluster categories of related objects like cats. Incidentally, immersions also arise in vision when a 3D object partially occludes itself from some viewpoint, hence its full 2D contour has visible and invisible parts which, together, form an immersed curve in the image plane. It is a central problem in computer vision to devise algorithms by which computers can similarly recognize and cluster shapes. Many types of metrics have been proposed for this purpose. For example, there are L^1-type metrics such as the area of the symmetric difference of the interiors of two shapes. And there are L^∞-type metrics such as the Hausdorff metric: the maximum distance of points on either shape from the points on the other or of points outside one shape from points outside the other. The starting point of this investigation was whether one could use the manifold structure on the space of shapes and define an L^2-type metric by introducing a Riemannian structure on the space. In particular, we study some Riemannian metrics on the space of regular smooth curves in the plane, viewed as the orbit space of maps from S^1 to the plane modulo the group of diffeomorphisms of S^1, acting as reparameterizations. In particular we investigate the metric for a constant $A > 0$:

$$G_c^A(h,k) := \int_{S^1} (1 + A\kappa_c(\theta)^2)\langle h(\theta), k(\theta)\rangle |c'(\theta)|\, d\theta$$

where κ_c is the curvature of the curve c and h, k are normal vector fields to c. The term $A\kappa^2$ is a sort of geometric Tikhonov regularization because, for $A = 0$, the geodesic distance between any 2 distinct curves is 0, while for $A > 0$ the distance is always positive. We give some lower bounds for the distance function, derive the geodesic equation and the sectional curvature, solve the geodesic equation with simple endpoints numerically, and pose some open questions. The space has an interesting split personality: among large smooth curves, all its sectional curvatures are ≥ 0, while for curves with high curvature or perturbations of high frequency, the curvatures are ≤ 0.

The paper [M102] extends the result of [M98] that the L^2-Riemannian metric on the space of all immersions $S^1 \to \mathbb{R}^2$ which is invariant under the group $\operatorname{Diff}(S^1)$ induces vanishing geodesic distance on the quotient space $\operatorname{Imm}(S^1, \mathbb{R}^2)/\operatorname{Diff}(S^1)$, to the general high dimensional situation $\operatorname{Imm}(M, N)/\operatorname{Diff}(M)$ for any compact manifold M and Riemannian manifold (N, g) with $\dim N > \dim M$. On the open subset $\operatorname{Emb}(M, N)/\operatorname{Diff}(M)$ of all submanifolds of diffeomorphism type M in N (the non-linear Grassmanian) this says that the infinite dimensional analogon of the Fubini Study metric induces vanishing geodesic distance. We also carry over to the general case the metric from [M98] involving the second fundamental form. It turns out that we have only to use the mean curvature in order to get a good topological metric on the space $\operatorname{Imm}(S^1, \mathbb{R}^2)/\operatorname{Diff}(S^1)$ which is induced by the metric

$$G_f^A(h, k) := \int_M (1 + A\| \operatorname{Tr}^{f^*g}(S^f)\|^2_{g^{N(f)}}) g(h, k) \operatorname{vol}(f^*g)$$

where S^f is the second fundamental form of the immersion f. We also compute the sectional curvature of the L^2-metric in the hope to relate the vanishing of the geodesic distance to unbounded positivity of the sectional curvature: by going through ever more positively curved parts of the space we can find ever shorter curves between any two submanifolds. The discussion of this is unfinished. In the final section of [M102] we show that the vanishing of the geodesic distance occurs even on the Lie group of all diffeomorphisms on each connected Riemannian manifold. We compute the sectional curvature also in this case.

In [H5] we considered a closed Riemannian manifold and the group of exact volume preserving diffeomorphisms as subgroup of the group of all volume preserving diffeomorphisms. We described all Riemannian manifolds for which this subgroup is totally geodesic. It turned out that the flat torus is essentially the only Riemannian manifold enjoying this property. More precisely, a Riemannian manifold which has this property is a twisted product of a flat torus and a Riemannian manifold with vanishing first Betti number (in which case these two groups coincide). In the second part we considered the same question for the group of Hamiltonian diffeomorphisms as subgroup of the group of symplectic diffeomorphisms. In the Kähler case we got the same answer. A closed Kähler manifold for which the group of Hamiltonian diffeomorphisms is a totally geodesic subgroup in the group of symplectic diffeomorphisms has to be a twisted product of a flat torus with a Kähler manifold with vanishing first Betti number.

In [H8] we studied the question of perfectness of the group of diffeomorphisms. Results of Hermann, Thurston, Mather and Epstein tell that the connected component of the group of diffeomorphisms is a perfect (actually simple) group, i.e. every diffeomorphism can be written as a product of commutators. Our aim was to show that it is possible to choose the factors smoothly, i.e. solve the equation $f = [g_1, h_1] \circ \cdots \circ [g_N, h_N]$ for given diffeomorphisms f in a way so that the diffeomorphisms g_i and h_i depend smoothly on f. For a class of manifolds, including odd dimensional spheres and compact Lie groups, we obtained an affirmative answer and estimates on how many factors are necessary for f sufficiently close to the identity. This is a consequence of another theorem we have proved in [H8]: Whenever a compact manifold is the total space of k fiber bundles so that all the vertical distributions together span the tangent bundle, then every diffeomorphism sufficiently close to the identity can be written as a product of a diffeomorphism $f_1 \cdots f_k$, where every f_i preserves the fibers of the i-th bundle.

In [H16] we continued the work started in [H8]. We managed to replace the Nash–Moser implicite function theorem used in [H8] with an elementary argument that works for non-compact manifolds as well. The main result we obtained reads as follows. For every manifold of dimension at least 2 one can choose the factors in a commutator presentation of a diffeomorphism $f = [g_1, h_1] \circ \cdots \circ [g_N, h_N]$ to depend smoothly on f and one has an estimate for N which grows linearly with the dimension of the underlying manifold, provided f is sufficiently close to the identity. For the sphere or the Euclidean space one can even take $N = 18$, independent of their dimension. This also provides a new proof of the simplicity of the diffeomorphism group, which too is based on Herman's result for the torus, but otherwise elementary.

In [H11] we considered the non-linear Grassmannian of n dimensional (unparameterized) closed oriented submanifolds of a given manifold M. If M is equipped with a closed $n+2$ form the Grassmannian inherits a (weak) symplectic structure. The main theorem of the paper is that in the case that the $n+2$ form represented an integral cohomology class the Grassmannian is pre-quantizeable in the sense that its symplectic form is the curvature of a principal S^1 bundle. This generalizes a theorem of Ismagilov who considered codimension two submanifolds. This has applications to central extensions of certain diffeomorphism groups and the realization of certain infinite dimensional Grassmannians as coadjoint orbits.

In the paper [M95] it is shown that continuous and smooth homotopies agree from smooth finite dimensional manifolds into infinite dimensional ones which are modeled on convenient vector spaces, in the sense of [G]. Since convex charts do not exist we use radial charts.

In the paper [M101] we construct a central extension by $\mathbb{R}$ of a group of diffeomorphisms of a manifold M with an exact 2-form ω_M and give conditions of its triviality. When $H^1(M, \mathbb{R}) = 0$ we prove that this extension is non-split for a manifold $\mathbb{R}^2 \times M$ with a form $\omega = \omega_0 + \omega_M$, where ω_0 is the standard symplectic form on $\mathbb{R}^2$, of the group of diffeomorphisms of $\mathbb{R}^2 \times M$ preserving the form ω. A little more detail: In his work on geometric quantization, Kostant defined a central extension of a group of diffeomorphisms of a manifold M respecting a closed integral 2-form ω, with kernel the real one dimensional torus. This extension is described as a group of automorphisms of a complex line bundle over M. In this paper we consider the case when the form ω is exact and $H^1(M, \mathbb{R}) = 0$. Using a construction described by Losik, we define a 2-cocycle $C(G, \omega)$ on a group G of diffeomorphisms of M preserving the form ω which gives a central extension $E(G, \omega)$ of the group G by $\mathbb{R}$. The geometric interpretation of this extension as a group of automorphisms of the trivial principal $\mathbb{R}$-bundle $M \times \mathbb{R}$ with a connection gives an equivalent definition of the extension $E(G, \omega)$ which is similar to that of the Kostant extension. Since we work with 2-cocycles we can define a smooth structure on our extension, describe conditions of when the cocycle $C(G, \omega)$ is trivial, and we can indicate the cases when this cocycle is nontrivial, i.e., the corresponding extension is non-split. In particular, we prove that it is non-split when $G = \mathrm{PSL}(2, \mathbb{R})$ is the group of orientation preserving isometries of the hyperbolic plane and ω is the area form, and the extension is related to the Steinberg extension. Since we work with 2-cocycles we can define a smooth structure on our extension, describe conditions of when the cocycle $C(G, \omega)$ is trivial, and we can indicate the cases when this cocycle is nontrivial, i.e., the corresponding extension is non-split. In particular, we prove that it is non-split when $G = \mathrm{PSL}(2, \mathbb{R})$ is the group of orientation preserving isometries of the hyperbolic plane and ω is the area form. This allows us to prove that the above extension is non-split for a manifold $\mathbb{R}^2 \times M$ with form $\omega = \omega_0 + \omega_M$, where ω_0 is the standard symplectic form on $\mathbb{R}^2$, and where ω_M is an exact form on M and the group $\mathrm{Diff}(\mathbb{R}^2 \times M, \omega)$ of diffeomorphisms of $\mathbb{R}^2 \times M$ preserving the form ω, if $H^1(M, \mathbb{R}) = 0$.

Spectral geometry. In [H6] we introduce a numerical invariant ρ associated to a closed one form ω and a Riemannian metric g. If this invariant is finite we showed how to improve Morse–Novikov theory in the sense that the incidence numbers (which are elements of a Novikov field) actually belong to a much smaller subring of holomorphic functions on the half plane $\Re(z) > \rho$. In this case we showed how to recover all the incidence numbers from spectral geometry of (ω, g). For this we generalized Witten–Helffer–Sjöstrand theory to closed one forms.

In [H12] we consider a system (M, g, ω, X) where (M, g) is a closed Riemannian manifold, ω a closed one form whose zeros are all non-degenerate and X an $(-\omega, g)$ gradient like vector field. For $t \in \mathbb{R}_+$ sufficiently large the Witten complex $(\Omega^*, d_t^* = d^* + t\omega \wedge \cdot)$ associated with (ω, g) decomposes canonically as a direct sum of two one parameter families of cochain complexes $(\Omega_{\mathrm{sm}}(t), d_t^*)$ and $(\Omega_{\mathrm{la}}(t), d_t^*)$. We show that under a mild hypotheses, generically satisfied, the inverse Laplace transform of $(\Omega_{\mathrm{sm}}(t), d^*(t))$ determines completely Novikov's counting of the instantons of the vector field X, organized as a Dirichlet series. The inverse Laplace transform of the analytic torsion of $(\Omega_{\mathrm{la}}(t), d^*(t))$ properly corrected, determines the counting function of closed trajectories of X.

In [H14] we extend, and Poincaré dualize, the concept of Euler structures, introduced by Turaev for manifolds with vanishing Euler–Poincaré characteristic, to arbitrary manifolds. We use the Poincaré dual concept, co-Euler structures, to remove all geometric ambiguities from the Ray–Singer torsion by providing a slightly modified object which is a topological invariant. We show that the modified Ray–Singer torsion when regarded as a function on the variety of complex representations of the fundamental group of the manifold is actually the absolute value of a rational function which we call in this paper the Milnor–Turaev torsion. As an example we provide a computation of this function for mapping tori.

In [H15] we provide a generalization of the Bismut–Zhang theorem to Morse–Bott–Smale vector fields. More precisely, given a closed Riemannian manifold and a Morse–Bott–Smale vector field, the Ray–Singer torsion can be computed as the sum of (1) the Ray–Singer torsions of the critical manifolds (these terms do not show up in the classical Bismut–Zhang theorem, since the critical manifolds are points in the Morse case), (2) a combinatorial torsion obtained from the finite dimensional spectral sequence associated with filtration on the Morse–Bott–Smale complex (generalizing the torsion of the Morse complex), and (3) a metric correction term. We consider this as a localization theorem for the Ray–Singer torsion. As an application we recover (a slight generalization of) a theorem of Lück, Schick and Thielmann who computed the torsion of the total space of a fiber bundle. This is derived from our theorem by noticing that a Morse–Smale function on the base, when pulled back to the total space, will be a Morse–Bott–Smale function. The paper also contains a detailed discussion of the compactification theorems for trajectory spaces and unstable manifolds in the Morse–Bott–Smale case.

Let me mention in this section where only Stefan Haller and his collaborators contributed, that the 'Habilitation' of Stefan Haller is in the final stages and should be decided soon.

Non-commutative geometry. About the paper [M86]: The noncommutative torus in its topological version (C^*-completion) as well as in its smooth version is one of the most important examples in noncommutative geometry. Beside the fact that the classical tools of differential geometry have unambiguous generalizations to it, it provides a very nontrivial example of noncommutative geometry. We looked at its smooth version and asked for its universal covering. We found the Heisenberg plane as it is presented in this paper: a twisted convolution on a carefully chosen space of distributions, namely the topological dual space $\mathcal{O}'_M$ of the Schwartz space $\mathcal{O}_M$ of smooth slowly increasing functions at ∞. It is large enough to contain the space of rapidly decreasing measures with support in the lattice $(2\pi\mathbb{Z})^2$ that is a space isomorphic to the space of smooth functions on the noncommutative torus (as well as on the usual commutative torus). The multiplication turns out to be a smooth curve in the deformation parameter $\hbar$. Moreover, looking at it via Fourier transform, Taylor expansion of the multiplication in the deformation parameter $\hbar$ leads to the formal Moyal star-product which is well known from deformation quantization. Then we noticed that we found examples of noncommutative $*$-algebras generalizing algebras of complex smooth functions. These $*$-algebras which can be realized as $*$-algebras of unbounded operators in Hilbert space admit "many" derivations specifying thereby the generalized smooth structure (see below). These algebras are tentatively called smooth $*$-algebras. An appendix gives an overview on convenient calculus in infinite dimensions (see [G]) which is necessary to obtain our results about smoothness in the deformation parameter $\hbar$, and which also gives the right setting for multilinear algebra with locally convex vector spaces. Work on this paper started in 1996, but we were unable to prove that the Heisenberg plane is a smooth $*$-algebra. Finally we gave up and stated this as a conjecture. The problem is finding enough states.

How much of the work promised in the application has been achieved

Many results in directions not foreseen in the application have been achieved, mainly in the directions **symplectic manifolds**, **spectral geometry**, and **Lie theory and representation theory**.

On the other hand, in some directions sketched in the application there were no results. The explanation is that Josef Teichmann took a position at TU Vienna after working as Post-Doc for two months only in this project. I should be mentioned here that Josef Teichmann (who worked in the predecessor of this project) got his 'Habilitation' for Mathematics at TU Vienna in the year 2002.

In the project application investigations in the following fields were promised:

Invariant Theory. This has been extremely successful. Papers [M91], [M89], [M99], [M105], [R], [M88], [M90], [M93], and [M94] contribute to this. A PhD thesis has been written in this field.

Completely integrable systems and double Lie groups. [M85], [Ho], and partly [M100] contribute to this.

Geodesics on infinite dimensional Lie groups and completely integrable systems. This was also extremely successful, but in an unexpected direction: We are very content about the work with D. Mumford, [M98] and [M102].

Further investigations of infinite dimen. regular Lie groups. The papers [H5], [H8], [H16], [H11], [M101] contribute to this subject. This is very successful work, but the questions sketched in the application (We want to answer the question, whether there exist non-regular convenient or even Fréchet-Lie-groups) remained unanswered.

Attempts for a structure theory of the Lie algebra of vector fields on a finite dimensional Lie group. No progress in this direction.

Approximation procedures on regular Fréchet Lie groups aiming towards solving certain non-linear partial differential equations. No progress in this direction.

A non-linear version of Arzelà-Ascoli's theorem on convenient Lie groups. No progress in this direction.

Actions of finite dimensional Lie groups and structures of orbit spaces. This was quite successful. Papers [M97], [M85], and [Ho] contribute.

Actions of Lie algebras on manifolds. Here the promised results have been delivered beyond expectation. [M83], [M92], [M96], and [M104] contain a rather complete answer.

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